Q: What is the probability that a particle in the ground state of the quantum harmonic oscillator

$$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \exp\left(-\frac{m\omega x^2}{2\hbar}\right) \tag{1}$$

will be observed outside the classically allowed region?

A: The potential of the classical harmonic oscillator (like its quantum analog) is

$$V(x) = \frac{1}{2} m\omega^2 x^2 \tag{2}$$

Setting this equal to the total energy of the ground state

$$E_0 = \frac{\hbar\omega}{2} \tag{3}$$

and solving for x results in the classical amplitude

$$A = \sqrt{\frac{\hbar}{m\omega}} \tag{4}$$

The probability we're after is

$$P(|x| > A) = 2\int_A^\infty |\psi|^2 \, \mathrm{d}x \tag{5}$$

Substituting (1) gives

$$= 2\sqrt{\frac{m\omega}{\pi\hbar}} \int_A^\infty \exp\left(-\frac{m\omega x^2}{\hbar}\right) \mathrm{d}x \tag{6}$$

or

$$= \frac{2}{A\sqrt{\pi}} \int_A^\infty e^{-x^2/A^2} \, \mathrm{d}x \tag{7}$$

This integral can be computed using numerical techniques. The result, which is independent of A, is approximately **16%**.